Cascade approach to current fluctuations in a chaotic cavity

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We propose a simple semiclassical method for calculating higher-order cumulants of current in multichannel mesoscopic conductors. To demonstrate its efficiency, we calculate the third and fourth cumulants of current for a chaotic cavity with multichannel leads of arbitrary transparency and compare the results with ensemble-averaged quantum-mechanical quantities. We also explain the discrepancy between the quantum-mechanical results and previous semiclassical calculations.

I. INTRODUCTION

In the last decade, there has been a large interest in current correlations in mesoscopic conductors.[1] Recently, also higher cumulants of current have received a significant attention of theorists. In a series of papers[2–4] a scattering approach to the distribution of charge transmitted through an arbitrary multi-terminal, multi-mode mesoscopic conductor, i.e. the so-called full counting statistics has been developed. As was shown in Ref.[3], the ensemble-averaged cumulants of arbitrary order can be calculated for any two-terminal conductor where the distribution of the transmission eigenvalues is known, e.g. for diffusive wires[5], chaotic cavities,[6–8], double-barrier tunnel junctions[10], or combinations of different conductors.[9]

Recently, Nazarov[11] presented a method for calculating the full counting statistics of charge transfer in conductors with a large number of quantum channels based on equations for the semiclassical Keldysh Green's functions. Subsequently, this method was extended[12] to multiterminal systems. Also, other approaches to higher cumulants, such as the nonlinear sigma model[13] for diffusive wires, have been proposed.

Common to all approaches[2–4,11–13] is that they are based on a quantum mechanical formulation. To obtain the cumulants for semiclassical systems, i.e. systems much larger than the Fermi wavelength, an ensemble average is performed and the number of transport modes is set to infinity, i.e. single-mode weak-localization-like corrections are neglected. Therefore it is of interest to have a completely semiclassical theory for the higher cumulants, which does not involve any quantum-mechanical quantities.

A step in this direction was made by de Jong[10], who calculated the distribution of charge transmitted through a double-barrier tunnel junction by applying a master equation to the transport in each completely independent transverse quantum channel. The results were in agreement with the quantum-mechanical theory in the limit of large channel number.

An attempt to construct a fully semiclassical theory of higher cumulants of current in a chaotic cavity was made by Blanter, Schomerus, and Beenakker[14] based on the principle of *minimal correlations*.[15] According to this principle, the fluctuations of the semiclassical distribution function of electrons in the cavity and the fluctuations of outgoing currents are related only through the condition of electron-number conservation, which is equivalent to the dephasing-voltage-probe approach[17] in quantum mechanics. However an attempt to extend the minimal-correlation approach to the fourth cumulant has led to a discrepancy with quantum-mechanical results,[15] which highly surprised the authors.[16]

Meanwhile the correlations imposed by the particle-number conservation are not the only possible ones in semiclassics. Quite recently the semiclassical Boltzmann–Langevin approach[18] has been extended to higher cumulants.[19] This extension takes into account the effect of lower cumulants on higher cumulants through the fluctuations of the distribution function and therefore it was termed *cascade* approach. Its equivalence with quantum-mechanical results[3] has been proven for diffusive metallic conductors.[19] In this paper, we show that the cascade approach is not restricted to diffusive metals or to conductors where the scattering is described by a collision integral, but it may be also applied to other systems that allow a semiclassical description, e.g. to chaotic cavities. To this end, we semiclassically calculate the third and fourth cumulants of current in a chaotic cavity taking into account cascade correlations and show that these values coincide with ensemble-averaged quantum-mechanical results.

The paper is organized as follows. In Section II we describe the model of chaotic cavity to be considered. The minimal-correlation results for the second cumulant of current are presented in Section III. In Section IV we

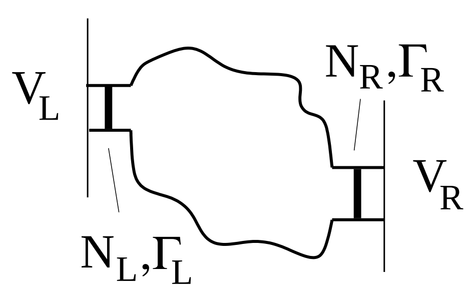

FIG. 1. A chaotic cavity with imperfect leads

calculate the third and fourth cumulants of current by means of the cascade approach. In Section V, the third cumulant of current is calculated by means of the quantum-mechanical circuit theory and its equivalence with the cascade results is shown. Section VI presents a conclusion where the results are summarized.

II. THE MODEL

Consider a chaotic cavity with two contacts of arbitrary transparency. The left contact has $N_L \gg 1$ channels and transparency Γ_L, and the right contact has N_R channels and a transparency Γ_R. The conductances of the leads $G_L = (e^2/\pi\hbar)N_L\Gamma_L$ and $G_R = (e^2/\pi\hbar)N_R\Gamma_R$ are also assumed to be much larger than $e^2/\hbar$, and the total conductance of the system is that of two resistors connected in series[20]

$$G = \frac{G_L G_R}{G_L + G_R}.$$

Because of strong chaotic scattering in the cavity, the electrons entering the cavity lose the memory of their phase on the time scale of the order of the time of flight through the cavity yet retain their energy. Therefore despite the quantum nature of the contacts, the cavity is a semiclassical object in the sense that the electrons inside it may be described by a semiclassical distribution function that depends only on the electron energy. Its average value is given by an expression[21]

$$f(\varepsilon) = \frac{G_L f_L(\varepsilon) + G_R f(\varepsilon)}{G_L + G_R}, \qquad (1)$$

where $f_L(\varepsilon)$ and $f_R(\varepsilon)$ are the distribution functions in the left and right electrodes.

III. THE PRINCIPLE OF MINIMAL CORRELATIONS

If the distribution function in the cavity were not allowed to fluctuate, the cavity could be considered just as a reservoir with non-Fermian distribution of electrons. The contacts would be independent generators of current noise and the cumulants of the corresponding extraneous noise currents could be obtained by differentiating the corresponding quantum-mechanical characteristic functions[22] for the charge transmitted in time t

$$\chi_{L,R}(\lambda, t) = \exp\Big\{ \frac{tN_{L,R}}{2\pi\hbar} \int d\varepsilon \ln\big\{1$$
$$+\Gamma_{L,R} f(\varepsilon)[1 - f_{L,R}(\varepsilon)](e^{i\lambda} - 1)$$
$$+ \Gamma_{L,R} f_{L,R}(\varepsilon)[1 - f(\varepsilon)](e^{-i\lambda} - 1)\big\}\Big\} \qquad (2)$$

with respect to the parameter λ the corresponding number of times.

In what follows, we will be interested only in the Fourier transforms of the current cumulants in the low-frequency limit and it will be implied that all the subsequent equations contain only low-frequency Fourier transforms of the corresponding quantities. Equation (2) leads to the following expressions for the cumulants of the noise current generated by the contacts:

$$\langle\langle \tilde{I}^n_{L,R} \rangle\rangle = \int d\varepsilon\, \langle\langle \tilde{I}^n_{L,R} \rangle\rangle_\varepsilon, \qquad (3)$$

where

$$\langle\langle \tilde{I}^2_{L,R} \rangle\rangle_\varepsilon = G_{L,R}[f_{L,R}(1-f) + f(1-f_{L,R})$$
$$- \Gamma_{L,R}(f_{L,R} - f)^2], \qquad (4)$$

$$\langle\langle \tilde{I}^3_{L,R} \rangle\rangle_\varepsilon = eG_{L,R}(f - f_{L,R})\{1 - 3\Gamma_{L,R}[f(1-f_{L,R})$$
$$+ f_{L,R}(1-f)] + 2\Gamma^2_{L,R}(f - f_L)^2\}, \qquad (5)$$

and

$$\langle\langle \tilde{I}^4_{L,R} \rangle\rangle_\varepsilon = e^2 G_{L,R}\Big\{ f_{L,R}(1-f) + f(1-f_{L,R})$$
$$+\Gamma_{L,R}(12 f^2_{L,R} f + 12 f^2 f_{L,R} - 12 f^2_{L,R} f^2$$
$$-7 f^2_{L,R} - 7f^2 + 2 f f_{L,R})$$
$$+12\Gamma^2_{L,R}(f_{L,R} - f)^2[f_{L,R}(1-f) + f(1-f_{L,R})]$$
$$- 6\Gamma^3_{L,R}(f_{L,R} - f)^4\Big\}. \qquad (6)$$

Since we are interested here only in low-frequency fluctuations, the pile-up of electrons in the cavity is forbidden. On the other hand, the noise currents $\tilde{I}_L$ and $\tilde{I}_R$ are absolutely independent, which would apparently result in a violation of the current-conservation law if these were the only contributions to the current noise. To ensure the current conservation at low frequencies, one has to take into account fluctuations of the distribution function $\delta f(\varepsilon)$ in the cavity.[21] Now the fluctuations of the current outgoing from the cavity to the left and right electrodes assume a form of Langevin equations, where $\tilde{I}_L$ and $\tilde{I}_R$ play the role of extraneous sources.

$$\delta I_{L,R} = \tilde{I}_{L,R} + \frac{1}{e} G_{L,R} \int d\varepsilon \delta f(\varepsilon). \qquad (7)$$

Extracting δf from the condition of current conservation

$$\delta I_L + \delta I_R = 0,$$

one obtains

$$\delta I_L = \frac{G_R \tilde{I}_L - G_L \tilde{I}_R}{G_L + G_R}. \tag{8}$$

By squaring this equation and using the independence of $\tilde{I}_L$ and $\tilde{I}_R$, one easily obtains that the second cumulant of the measurable current is

$$\langle\langle I_L^2 \rangle\rangle = \frac{G_R^2 \langle\langle \tilde{I}_L^2 \rangle\rangle + G_L^2 \langle\langle \tilde{I}_R^2 \rangle\rangle}{(G_L + G_R)^2}. \tag{9}$$

In the zero-temperature limit it gives

$$\langle\langle I_L^2 \rangle\rangle = eI\big[G_L G_R (G_L + G_L) + G_L^3 (1 - \Gamma_R) + G_R^3 (1 - \Gamma_L)\big] / (G_L + G_R)^3 \,, \tag{10}$$

where I is the average current flowing through the cavity. In the high-transparency limit $\Gamma_L = \Gamma_R = 1$ it reproduces the expression obtained by Blanter and Sukhorukov by means of the minimal-correlation principle and the exact quantum-mechanical results.

IV. CASCADE CORRECTIONS

A straightforward extension of the minimal correlation approach to higher cumulants has led to a discrepancy with the quantum mechanical results.[14] The reason is that the cavity is not just a reservoir with a nonequlibrium distribution of electrons. As suggested by Eqs. (7), their distribution function $f(\varepsilon)$ also exhibits fluctuations. As the cumulants of the currents $\tilde{I}_L$ and $\tilde{I}_R$ are functionals of the distribution function in the cavity, its fluctuation δf changes them too. Since the characteristic time scale for δf is of the order of the dwell time of an electron in the cavity, these changes are slow on the scale of the correlation time of extraneous currents, and therefore the cumulants of these currents adiabatically follow δf. This results in additional correlations, which may be termed "cascade" because lower-order correlators of extraneous currents contribute to higher-order cumulants of measurable quantities. One can write for the low-frequency transforms of the corresponding quantities

$$\delta\langle\langle \tilde{I}_{L,R}^n \rangle\rangle = \int d\varepsilon \, \frac{\delta\langle\langle \tilde{I}_{L,R}^n \rangle\rangle}{\delta f(\varepsilon)} \delta f(\varepsilon), \tag{11}$$

where $\delta\langle\ldots\rangle/\delta f$ denotes a functional derivative of the corresponding quantity with respect to $f(\varepsilon)$. For example, the third cumulant of the current may be written as the sum of the minimal-correlation value

$$\langle\langle I_L^3 \rangle\rangle_m = \frac{G_R^3 \langle\langle \tilde{I}_L^3 \rangle\rangle - G_L^3 \langle\langle \tilde{I}_R^3 \rangle\rangle}{(G_L + G_R)^3} \tag{12}$$

and the cascade correction

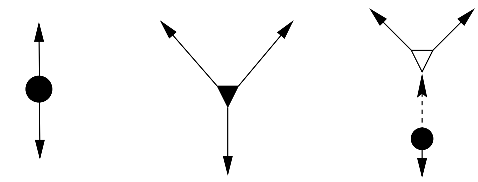

FIG. 2. The second cumulant and the two contributions to the third cumulant of current. The external ends correspond to current fluctuations at different moments of time and the dashed lines, to fluctuations of the distribution function. The full circle and triangle correspond to cumulants of extraneous currents and the empty triangle to the functional derivative of the second cumulant.

$$\Delta\langle\langle I_L^3 \rangle\rangle = 3 \int d\varepsilon \, \frac{\delta\langle\langle I_L^2 \rangle\rangle}{\delta f(\varepsilon)} \langle \delta f(\varepsilon) \delta I_L \rangle. \tag{13}$$

The factor 3 is due to the fact that this equation in general includes three different currents and allows three inequivalent permutations of them.

The cascade corrections are conveniently presented in a diagrammatic form[19] (see Figs. 2 and 3). The rules for constructing these diagrams strongly differ from the ones known for Green's functions in quantum mechanics. The diagrams do not present an expansion in any small parameter and their number is strictly limited for a cumulant of a given order. All diagrams present graphs, whose outer vertices correspond to different instances of current and whose inner vertices correspond either to cumulants of extraneous currents or their functional derivatives. The number of arrows outgoing from an inner vertex corresponds to the order of the cumulant and the number of incoming arrows corresponds to the order of a functional derivative. Since the nth cumulant presents a polynomial of the distribution function of degree n, the number of incoming arrows at any inner vertex cannot exceed the number of outgoing arrows. Apparently, the difference between the total order of cumulants involved and the total number of functional differentiations should be equal to the order of the cumulant being calculated. As there should be no back-action of higher cumulants on lower cumulants, all diagrams are singly connected. Therefore any diagram for the nth cumulant of the current may be obtained from a diagram of order $m < n$ by combining it with a diagram of order $n - m + 1$, i.e. by inserting one of its outer vertices into one of the inner vertices of the latter. Hence the most convenient way to draw diagrams for a cumulant of a given order is to start with diagrams of lower order and to consider all their inequivalent combinations that give diagrams of the desired order. The analytical expressions corresponding to each diagram contain numerical prefactors equal to the numbers of inequivalent permutations of the outer vertices.

Unlike the case of a diffusive conductor, the third and fourth cumulants include now *all* possible diagrams and

not only those that are constructed of second-order cumulants (Fig. 2, diagram a). The third cumulant is presented by diagrams b and c in Fig. 2. Diagram b presents the minimal-correlation value (12) and diagram c presents the only possible cascade correction (13) obtained by combining two second cumulants.

We are now in position to evaluate the diagrams. The functional derivative is easily obtained by differentiating Eq. (4) and substituting it into (9), which gives

$$\frac{\delta\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon)} = \frac{G_L G_R}{(G_L+G_R)^2}\Big\{G_L\big[1-2f_R+2\Gamma_R(f_R-f)\big] + G_R\big[1-2f_L+2\Gamma_L(f_L-f)\big]\Big\}. \tag{14}$$

To calculate the fluctuation δf, one has to write down equations (7) and the current-conservation law in the energy-resolved form. This immediately gives

$$\delta f(\varepsilon) = -\frac{e}{G_L+G_R}\left[(\tilde{I}_L)_\varepsilon + (\tilde{I}_R)_\varepsilon\right] \tag{15}$$

where $(\tilde{I}_{L,R})_\varepsilon$ are energy-resolved extraneous currents. Since fluctuations at different energies are completely independent, one easily obtains that

$$\langle\delta f(\varepsilon)\delta I_L\rangle = \frac{e}{(G_L+G_R)^2}\left[G_L\langle\langle\tilde{I}_R^2\rangle\rangle_\varepsilon - G_R\langle\langle\tilde{I}_L^2\rangle\rangle_\varepsilon\right]. \tag{16}$$

Hence the total third cumulant, which is the sum of (12) and (13), is of the form

$$\langle\langle I_L^3\rangle\rangle = -\frac{e^2 I}{(G_L+G_R)^6}\Big\{(G_L+G_R)\big[(G_L+G_R)^2 \times(G_L^3+G_R^3) - 3(G_L+G_R)(\Gamma_L G_R^4+\Gamma_R G_L^4) + 2\Gamma_L^2 G_R^5 + 2\Gamma_R^2 G_L^5\big] - 3G_L G_R\big[G_L^2(1-\Gamma_R) - G_R^2(1-\Gamma_L)\big] \times\left[G_L^2(1-2\Gamma_R) - G_R^2(1-2\Gamma_L)\right]\Big\}. \tag{17}$$

In the case of perfectly transparent leads $\Gamma_L = \Gamma_R = 1$ the cascade correction to the third cumulant is zero and the minimal-correlation result

$$\langle\langle I_L^3\rangle\rangle = -e^2 I\frac{G_L G_R (G_L-G_R)^2}{(G_L+G_R)^4}$$

is reproduced. This is why the discrepancy between the minimal-correlation and quantum-mechanical results was noted by Blanter and co-workers only for the fourth cumulant.

The fourth cumulant is presented by a sum of six diagrams shown in Fig. 3. Diagram a presents the minimal-correlation value

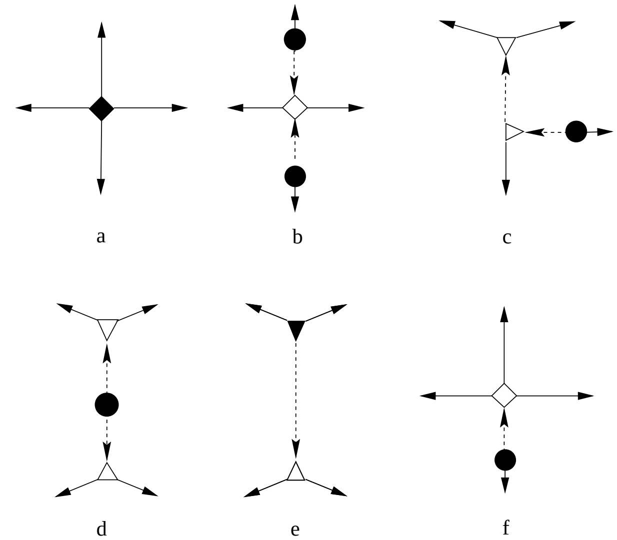

FIG. 3. The contributions to the fourth cumulant of the current. Dashed lines correspond to fluctuations of the distribution function. Full circles, triangles and squares correspond to the second, third, and fourth cumulants of extraneous currents. The empty triangles and squares present their functional derivatives.

$$\langle\langle I_L^4\rangle\rangle_m = \frac{G_R^4\langle\langle\tilde{I}_L^4\rangle\rangle + G_L^4\langle\langle\tilde{I}_R^4\rangle\rangle}{(G_L+G_R)^4}. \tag{18}$$

The rest of diagrams are obtained by combining the diagrams for the third and second cumulants. Diagrams b and c are obtained by inserting the second cumulant into the diagram c for the third cumulant, and diagram d is obtained by plugging diagram c for the third cumulant into the second cumulant. Diagrams e and f are obtained by inserting diagram b into diagram a and diagram a into diagram b. The corresponding analytical expressions contain numerical prefactors 1, 6, 12, 3, 6, and 4 that present the numbers of inequivalent permutations of four currents entering into the cumulant.

The first correction is given by an expression

$$\Delta_1\langle\langle I_L^4\rangle\rangle = 6\int d\varepsilon_1\int d\varepsilon_2\,\frac{\delta^2\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon_1)\delta f(\varepsilon_2)} \times\langle\delta f(\varepsilon_1)\delta I_L\rangle\langle\delta f(\varepsilon_2)\delta I_L\rangle. \tag{19}$$

The second functional derivative

$$\frac{\delta^2\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon_1)\delta f(\varepsilon_2)} = -2\delta(\varepsilon_1-\varepsilon_2)\frac{G_L G_R(G_R\Gamma_L+G_L\Gamma_R)}{(G_L+G_R)^2} \tag{20}$$

is obtained by differentiating (9) twice with respect to $f(\varepsilon)$, and the two correlators in (19) are given by (16).

The second cascade correction is given by an expression

$$\Delta_2\langle\langle I_L^4\rangle\rangle = 12\int d\varepsilon_1\,\frac{\delta\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon_1)}\int d\varepsilon_2\,\frac{\delta\langle\delta f(\varepsilon_1)\delta I_L\rangle}{\delta f(\varepsilon_2)}$$

$$\times \langle \delta f(\varepsilon_2)\delta I_L\rangle, \tag{21}$$

where the first functional derivative is given by (14),

$$\frac{\delta\langle \delta f(\varepsilon_1)\delta I_L\rangle}{\delta f(\varepsilon_2)} = 2\delta(\varepsilon_1-\varepsilon_2)\frac{G_L G_R}{(G_L+G_R)^2}\big[(\Gamma_L-\Gamma_R)f + (1-\Gamma_L)f_L - (1-\Gamma_R)f_R\big], \tag{22}$$

and the last correlator is given by (16).

The third contribution is given by an expression

$$\Delta_3\langle\langle I_L^4\rangle\rangle = 3\int d\varepsilon_1 \int d\varepsilon_2 \,\frac{\delta\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon_1)}\langle \delta f(\varepsilon_1)\delta f(\varepsilon_2)\rangle\frac{\delta\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon_2)}, \tag{23}$$

where the functional derivatives are given by (14) and the second cumulant of the distribution function

$$\langle \delta f(\varepsilon_1)\delta f(\varepsilon_2)\rangle = e^2\delta(\varepsilon_1-\varepsilon_2)\frac{\langle\langle \tilde{I}_L^2\rangle\rangle_{\varepsilon_1}+\langle\langle \tilde{I}_R^2\rangle\rangle_{\varepsilon_1}}{(G_L+G_R)^2} \tag{24}$$

is obtained by multiplying equations (15) with $\varepsilon = \varepsilon_1$ and $\varepsilon = \varepsilon_2$.

The fourth cascade correction involves third-order cumulants of extraneous currents and is given by an expression

$$\Delta_4\langle\langle I_L^4\rangle\rangle = 6\int d\varepsilon\,\frac{\delta\langle\langle I_L^2\rangle\rangle}{\delta f(\varepsilon)}\langle \delta f(\varepsilon)\delta I_L^2\rangle_m, \tag{25}$$

where

$$\langle \delta f(\varepsilon)\delta I_L^2\rangle_m = -e\frac{G_R^2\langle\langle \tilde{I}_L^3\rangle\rangle_\varepsilon + G_L^2\langle\langle \tilde{I}_R^3\rangle\rangle_\varepsilon}{(G_L+G_R)^3} \tag{26}$$

is obtained by multiplying one equation (15) and two equations (7) and averaging them with the correlators (5).

The fifth correction is given by

$$\Delta_5\langle\langle I_L^4\rangle\rangle = 4\int d\varepsilon\,\frac{\delta\langle\langle I_L^3\rangle\rangle_m}{\delta f(\varepsilon)}\langle \delta f(\varepsilon)\delta I_L\rangle. \tag{27}$$

The functional derivative in the integrand

$$\frac{\delta\langle\langle I_L^3\rangle\rangle_m}{\delta f(\varepsilon)} = e\frac{G_L G_R}{(G_L+G_R)^3}\Big\{G_R^2\big[1-6\Gamma_L f(1-f) -6\Gamma_L(1-\Gamma_L)(f-f_L)^2\big] - G_L^2\big[1-6\Gamma_R f(1-f)-6\Gamma_R(1-\Gamma_R)(f-f_L)^2\big]\Big\} \tag{28}$$

is calculated similarly to (14).

The total fourth cumulant of current is given by the sum of its minimal-correlation value (18) and the cascade corrections given by (19), (21), (23), (25), and (27). The full resulting expression of rather complicated form is given in the Appendix, and here we give only its limiting values

$$\langle\langle I_L^4\rangle\rangle = e^3 V\frac{G_L^2 G_R^2}{(G_L+G_R)^7}\big[G_L^4 - 8G_R G_L^3 + 12G_L^2G_R^2 - 8G_L G_R^3 + G_R^4\big] \tag{29}$$

in the high-transparency limit $\Gamma_L = \Gamma_R = 1$ and

$$\langle\langle I_L^4\rangle\rangle = e^3 V\frac{G_L G_R}{(G_L+G_R)^7}\big[G_L^6 - 8G_L^5 G_R + 31G_L^4G_R^2 - 40G_L^3G_R^3 + 31G_L^2G_R^4 - 8G_L G_R^5 + G_R^6\big] \tag{30}$$

in the low-transparency limit where $\Gamma_L \to 0$ and $\Gamma_R \to 0$.

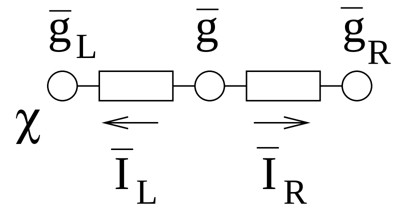

FIG. 4. The circuit theory representation of the junction, with matrix Greens functions $\bar{g}_L, \bar{g}_R$ and $\bar{g}$, matrix currents $\bar{I}_R$ and $\bar{I}_L$ and the counting field χ shown.

V. CIRCUIT-THEORY RESULTS

We now show that the same results for the cumulants can be obtained within an ensemble-averaged quantum mechanical Green's function approach. We use the circuit theory of full counting statistics, recently developed by Nazarov and Bagrets,[12] which allows us to calculate cumulant by cumulant in a systematic way. For shortness of the presentation, we only calculate the first three cumulants.

The circuit theory representation of the junction is shown in Fig 4. It consists of three "nodes", the two reservoirs and the dot, connected via two "resistances", the left and right point contact. Each node is assigned a 2×2 matrix Green's function, i.e. $\bar{g}_L, \bar{g}_R$ and $\bar{g}$. The matrices are subjected to a normalization condition $\bar{g}_L^2 = \bar{g}_R^2 = \bar{g}^2 = 1$. The Green's functions of the two reservoirs are known,

$$\bar{g}_L = e^{i\chi\bar{\tau}_z/2}g_L e^{-i\chi\bar{\tau}_z/2},\ \bar{g}_R = g_R,$$
$$g_{L,R} = \begin{pmatrix} 1-2f_{L,R}(\varepsilon) & -2f_{L,R}(\varepsilon) \\ -2(1-f_{L,R}(\varepsilon)) & 2f_{L,R}(\varepsilon)-1\end{pmatrix}, \tag{31}$$

where $\bar{\tau}_z$ is the Pauli matrix, $f_{L,R}(\varepsilon)$ are the Fermi distribution function of the reservoirs and χ is the counting field (due to current conservation, it is only necessary to

count the electrons in one reservoir). The matrix $\bar{g}(\chi)$ is determined from a matrix current conservation equation

$$\bar{I}_L + \bar{I}_R = 0,\ \bar{I}_{L,R} = \frac{G_{L,R}[\bar{g}_{L,R}, \bar{g}]}{4 + \Gamma_{L,R}[\{\bar{g}_L, \bar{g}\} - 2]} \qquad (32)$$

where $[..,..]$ is the commutator and $\{..,..\}$ the anti-commutator. Knowing $g(\chi)$, the full counting statistics of charge transfer can be found. However, in the system under study, it is not possible to find an explicit expression for $g(\chi)$ in the general case (arbitrary N_L, N_R and Γ_L, Γ_R), and the full counting statistics has to be studied by numerical means. Here we are only interested in the first three cumulants, which can be found analytically by an expansion of the Green's functions in the counting field χ.

The first three cumulants are given by (evaluated at the left contact)

$$\begin{aligned} I &= \frac{e}{h}\int dE \ \mathrm{tr}\,[\bar{\tau}_z \bar{I}_L]\big|_{\chi=0} \\ \langle\langle I_L^2\rangle\rangle &= -i\frac{e^2}{h}\int dE \ \mathrm{tr}\left[\bar{\tau}_z \frac{d\bar{I}_L}{d\chi}\right]\bigg|_{\chi=0} \\ \langle\langle I_L^3\rangle\rangle &= -\frac{e^3}{h}\int dE \ \mathrm{tr}\left[\bar{\tau}_z \frac{d^2\bar{I}_L}{d\chi^2}\right]\bigg|_{\chi=0} \end{aligned} \qquad (33)$$

To calculate these cumulants, we thus need to expand the Green's functions, and correspondingly the matrix currents, to second order in the counting field χ, i.e.

$$\bar{g}(\chi) = \bar{g}^{(0)} + \chi\bar{g}^{(1)} + \frac{\chi^2}{2}\bar{g}^{(2)},\ \bar{g}^{(n)} \equiv \frac{d^n\bar{g}}{d\chi^n}\bigg|_{\chi=0} \qquad (34)$$

and similarly for the other quantities. For simplicity, we consider the case with zero temperature and the left reservoir held at a finite voltage eV. In this case, only the energies $0 < \varepsilon < eV$ need to be considered, where $f_L(\varepsilon) = 1$ and $f_R(\varepsilon) = 0$, and we drop the energy notation below.

For the first cumulant, we have the matrix currents to zeroth order in the counting field,

$$\bar{I}_{L,R}^{(0)} = \frac{G_{L,R}[\bar{g}_{L,R}^{(0)}, \bar{g}^{(0)}]}{4 + \Gamma_{L,R}[\{\bar{g}_{L,R}^{(0)}, \bar{g}^{(0)}\} - 2]}. \qquad (35)$$

where from Eq. (31), one has $\bar{g}_{R,L}^{(0)} = g_{R,L}$. From the matrix current equation in Eq. (32), i.e. $I_R^{(0)} + I_L^{(0)} = 0$, we then obtain

$$\bar{g}^{(0)} = \begin{pmatrix} 1-2f & -2f \\ -2(1-f) & 2f-1 \end{pmatrix},\ f = \frac{G_L}{G_L + G_R} \qquad (36)$$

where f is the distribution function in the dot, as in Eq. (1). Knowing $\bar{g}^{(0)}$ we find $\bar{I}_L^{(0)}$ from Eq. (35) and then the current from Eq. (33)

For the second cumulant, we need to expand the matrix currents to first order in χ. Noting that the expressions in the matrix denominator appearing in the expansion, is $4 + \Gamma_{L,R}[\{\bar{g}_{L,R}^{(0)}, \bar{g}^{(0)}\} - 2] = 4$, we obtain

$$\begin{aligned} \bar{I}_L^{(1)} &= \frac{G_L}{4}\left([\bar{g}_L^{(1)}, \bar{g}^{(0)}] + [\bar{g}_L^{(0)}, \bar{g}^{(1)}]\right) \\ &+ \frac{G_L}{16}\left(\{\bar{g}_L^{(1)}, \bar{g}^{(0)}\} + \{\bar{g}_L^{(0)}, \bar{g}^{(1)}\}\right), \\ \bar{I}_R^{(1)} &= \frac{G_R}{4}[\bar{g}_R^{(0)}, \bar{g}^{(1)}] + \frac{G_R}{16}\{\bar{g}_R^{(0)}, \bar{g}^{(1)}\}. \end{aligned} \qquad (37)$$

In addition to the matrix current equation $\bar{I}_L^{(1)} + \bar{I}_R^{(1)} = 0$, we get an extra condition for $\bar{g}^{(1)}$ from the normalization condition $\bar{g}(\chi)^2 = 1$, namely

$$\begin{aligned} \bar{g}^2(\chi) &= 1 + \chi\{\bar{g}^{(0)}, \bar{g}^{(1)}\} + O(\chi^2) = 1 \\ &\Rightarrow \{\bar{g}^{(0)}, \bar{g}^{(1)}\} = 0 \end{aligned} \qquad (38)$$

Staring from the ansatz (equivalent to the parametrization in Ref. 12)

$$\bar{g}^{(1)} = \begin{pmatrix} h_{11}^{(1)} & h_{12}^{(1)} \\ h_{21}^{(1)} & -h_{11}^{(1)} \end{pmatrix}, \qquad (39)$$

Eq. (38) gives $h_{21}^{(1)} = h_{11}^{(1)}(1-2f)/f - h_{12}^{(1)}(1-f)/f$. Inserting $\bar{g}^{(1)}$ into the matrix current equation $\bar{I}_L^{(1)} + \bar{I}_R^{(1)} = 0$ gives $h_{11}^{(1)} = h_{12}^{(1)} + 4f^2$ and then

$$\begin{aligned} h_{12}^{(1)} = -\frac{4G_L}{(G_L+G_R)^4}&\left[G_L^3 + G_L^2 G_R(1+\Gamma_R)\right. \\ &\left. + G_L G_R^2 + G_R^3(1-\Gamma_L)\right]. \end{aligned} \qquad (40)$$

From $h_{12}^{(1)}$ we thus obtain all components of $\bar{g}^{(1)}$. Inserting this into the matrix currents in Eq. (37) we get the second cumulant from Eq. (33). It coincides exactly with Eq. (10).

The calculation of the third cumulant proceeds along the same lines. One first expands the matrix currents to second order in χ (not presented due to the lengthy expressions). The requirement that the $O(\chi^2)$ term in Eq. (38) should be zero gives $\{\bar{g}^{(2)}, \bar{g}^{(0)}\} + 2(\bar{g}^{(1)})^2 = 0$. Using the ansatz

$$\bar{g}^{(2)} = \begin{pmatrix} h_{11}^{(2)} & h_{12}^{(2)} \\ h_{21}^{(2)} & -h_{11}^{(2)} \end{pmatrix}, \qquad (41)$$

one gets $h_{21}^{(2)} = \left[(g_a^{(1)})^2 + g_b^{(1)} g_c^{(1)}\right]/f + h_{11}^{(2)}(1-2f)/f - h_{12}^{(2)}(1-f)/f$. The matrix current equation $\bar{I}_L^{(2)} + \bar{I}_R^{(2)} = 0$ then gives $h_{11}^{(2)}$ and $h_{12}^{(2)}$ (not written out), which fully determines $\bar{g}^{(2)}$. Inserting this into the expression for the matrix currents we find the third cumulant from Eq. (33). It coincides exactly with Eq. (17).

We point out that it is in principle possible to obtain analytical expressions for all higher cumulants in the

same way, although the procedure is rather cumbersome already for the third cumulant.

The third and fourth cumulants of current may be also obtained by means of random-matrix theory[23] using the diagrammatic technique proposed by Brouwer and Beenakker.[8] Substituting the resulting transmission probabilities for the whole system and using Eq. (2), one obtains expressions that coincide with Eq. (17) and the expression for the fourth cumulant given in the Appendix.

VI. CONCLUSION

In summary, we have shown that the semiclassical cascade approach gives the same results for the third and fourth cumulants of current in a chaotic cavity with imperfect leads as the circuit theory. This leads us to the conclusion that this approach may be applied to a wide class of systems that may include both semiclassical and quantum-mechanical elements. The advantage of the cascade approach is its physical transparency and a relative simplicity. For example, if the system consists of a number of cavities connected by contacts whose cumulants of current are known, this approach allows one to easily construct the cumulants of the current for the whole system. In principle, it also allows an inclusion of inelastic scattering processes and a calculation of cross-correlated cumulants of current in multiterminal systems. Therefore it presents a reasonable alternative to the full counting statistics based on the circuit theory.

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APPENDIX

In the case of arbitrary transmissions of the contacts the fourth cumulant is given by the expression

$$
\begin{aligned}
\langle\langle I_L^4\rangle\rangle = -e^3 I\Big[&(\Gamma_R-1)(6\Gamma_R^2-6\Gamma_R+1)G_L^9\\
&+(60\Gamma_R^2-30\Gamma_R-36\Gamma_R^3+5)G_R G_L^8\\
&+(30\Gamma_R^3-10-60\Gamma_R^2+45\Gamma_R)G_R^2 G_L^7\\
&+(120\Gamma_L\Gamma_R^2-60\Gamma_R^2-30+92\Gamma_R+55\Gamma_L-168\Gamma_L\Gamma_R)G_R^3 G_L^6\\
&+(72\Gamma_L\Gamma_R+4+72\Gamma_R^2-6\Gamma_L-96\Gamma_L\Gamma_R^2-51\Gamma_R)G_R^4 G_L^5\\
&+(4-6\Gamma_R-51\Gamma_L+72\Gamma_L^2+72\Gamma_L\Gamma_R-96\Gamma_L^2\Gamma_R)G_R^5 G_L^4\\
&+(92\Gamma_L-60\Gamma_L^2-30-168\Gamma_L\Gamma_R+120\Gamma_L^2\Gamma_R+55\Gamma_R)G_R^6 G_L^3\\
&+(45\Gamma_L+30\Gamma_L^3-10-60\Gamma_L^2)G_R^7 G_L^2\\
&+(-36\Gamma_L^3-30\Gamma_L+5+60\Gamma_L^2)G_R^8 G_L\\
&+(-1+\Gamma_L)(6\Gamma_L^2-6\Gamma_L+1)G_R^9\Big]\Big/(G_L+G_R)^9.
\end{aligned}
$$

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